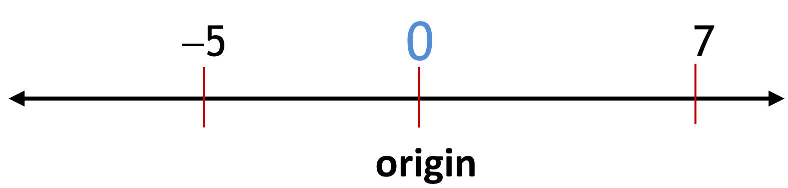
ABSOLUTE VALUE – THE BASICS

Sometimes the "size" of a number is more important than the "sign" of the number, that is, whether it's positive, zero, or negative. To measure the size of a number (without regard to its sign), we consider the ***absolute value*** of that number. There are two equivalent approaches to the meaning of absolute value.

❑ *TWO WAYS OF LOOKING AT ABSOLUTE VALUE*

1) **Geometry**

On a number line, the ***absolute value*** of a number (any kind of real number) is the ***distance*** from that number to **0** (the origin) on the line.



What is the distance from the number **7** to 0? It's 7. So we say that the absolute value of 7 is **7**.

How far from the origin is the number **–5**? It's 5 units away, and thus the absolute value of –5 is **5**.

What is the distance from the origin to $\mathbf{2\pi}$? The distance is 2π, and we conclude that the absolute value of 2π is $\mathbf{2\pi}$.

What is the distance between the number **0** and the origin? It's 0; conclusion: the absolute value of 0 is **<u>0</u>**.

Notes on Distance:

First, we can calculate the distance from <u>any</u> number to 0 (the origin). In other words, the distance between the origin and any point on the line <u>always</u> exists, even if it's hard or impossible to calculate.

Second, distance must be greater than or equal to 0; that is, it's <u>never</u> negative. If $\boldsymbol{d}$ represents distance, then $\boldsymbol{d} \geq \mathbf{0}$.

2) Arithmetic

If a number is positive, the *absolute value* of the number is the same number. For example, the absolute value of 9 is **<u>9</u>**.

If a number is negative, the *absolute value* of the number is the <u>opposite</u> of the number. For example, the absolute value of –5 is **<u>5</u>**.

And the *absolute value* of 0 is simply **<u>0</u>**.

Put more simply, the absolute of a number greater than or equal to 0 is the same number; if the number is negative, just remove the minus sign to calculate its absolute value.

❑ *NOTATION*

Instead of writing the words *absolute value* all the time, we do what we always do in math: condense the concept into symbols. To represent the absolute value of a number, we put vertical bars around the number. Thus,

In computers, the *absolute value* of *n* would be written

abs(*n*)

The absolute value of 9 is 9: $|9| = \mathbf{9}$

The absolute value of –5 is 5: $|-5| = \mathbf{5}$

The absolute value of 0 is 0: $|0| = \mathbf{0}$

Just for repetition: If a number is greater than or equal to 0, then its absolute value is that same number. If a number is less than 0 (which means it's a negative number), then its absolute value is the opposite of that number (which will then turn into a positive number). We can write this in the following way; it looks confusing, but it's totally valid:

If $x \ge 0$, then $|x| = x$

If $x < 0$, then $|x| = -x$

This definition ensures that the absolute value of a quantity is never negative.

Here are some more examples of absolute value:

$|17.5| = 17.5$ $\qquad |0| = 0$ $\qquad |-13| = 13$

$|\pi| = \pi$ $\qquad |-\sqrt{7}| = \sqrt{7}$ $\qquad |-3\pi| = 3\pi$

Bottom Line:

The *absolute value* of a quantity is either positive or zero:

If x is ANY quantity, then $|x| \ge 0$.

In other words, the absolute value of a quantity is never negative.

To illustrate this point, even though you may not be able to calculate

$$\left|\sin^2(\pi/6) - \ln(e-1)\right|$$

until Pre-calculus, you should still be able to understand that the answer to this problem — whatever it is — is greater than or equal to zero. Equivalently, the answer is not negative.

EXAMPLE 1: **Evaluate each expression:**

A. $|7-2| = |5| = \mathbf{5}$

B. $|3^2-15| = |9-15| = |-6| = \mathbf{6}$

C. $-|2-7| = -|-5| = \mathbf{-5}$

D. $|-3-4|-|10-2| = |-7|-|8| = 7-8 = \mathbf{-1}$

Homework

1. The absolute value of any number is ______.

2. If x represents any number, then

   a. $|x| > 0$ b. $|x| < 0$ c. $|x| \ge 0$ d. $|x| \le 0$

3. Which one of the following inequalities has NO solution?

   a. $|x| > 0$ b. $|x| < 0$ c. $|x| \ge 0$ d. $|x| \le 0$

4. True/False:

   a. Every number has an absolute value.
   b. There is a number whose absolute value is 0.
   c. There is a number whose absolute value is negative.
   d. There are two different numbers whose absolute value is 9.

5. Simplify each expression:

   a. $|4-14|$ b. $|2(-3)-7|$ c. $-|-9|$ d. $|2^0-\pi^0|$

6. Simplify each expression:

   a. $|7\pi|$ b. $|-7\pi|$ c. $|\sqrt{67}|$ d. $|-\sqrt{2.7}|$ e. $|0^{3\pi}|$

7. What's the best statement you can make regarding the expression

$$\left| \frac{\tan x - \sin x}{e^{\int \sec x \, dx}} \right| ?$$

8. Consider the statement: $|a \cdot b| = |a| \cdot |b|$

   a. Give three examples where the statement is true, using

      i) two positive numbers.
      ii) two negative numbers.
      iii) two numbers of opposite sign.

   b. Do you think the statement is true or false?

9. Consider the statement: $|x + y| = |x| + |y|$

   a. Give an example where the statement is true.

   b. Give an example where the statement is false.

   c. So, is the statement true or false?

10. Consider the statement: $|x + y| < |x| + |y|$

    a. Give an example where the statement is true.

    b. Give an example where the statement is false.

    c. Is the statement true or false?

11. [Hard] Try to use the previous two problems to come up with a statement that is TRUE.

The *absolute value* of a positive number is *itself*.

The *absolute value* of 0 is 0.

The *absolute value* of a negative number is its *opposite*.

Solutions

1. "greater than or equal to 0"
OR "never negative"
OR " ≥ 0 "

2. c.

3. b.

4. a. T b. T c. F d. T

5. a. 10 b. 13 c. -9 d. 0

6. a. 7π b. 7π c. $\sqrt{67}$ d. $\sqrt{2.7}$ e. 0

7. If you don't know enough algebra and trig, the best you could say is that, whatever legal value of x you place in the expression, the result WON'T BE NEGATIVE. That is, the expression represents something that is GREATER THAN OR EQUAL TO 0.

8. Although a whole slew of examples does not prove that the statement is true, this particular statement <u>is</u> always true.

9. Recall that if a statement fails in just one case, it's considered a false statement.

10. Again, if a statement fails in just one case, it's considered a false statement.

11. I will not give the answer away, but it's called the *Triangle Inequality*, and is one of the most important rules in math.

"Who questions much, shall learn much, and retain much."

– Francis Bacon